UCSD/PTH 94-26
hep-ph/9502311

SU(3) Corrections to $B \to Dl\overline{\nu}$ Form Factors at $\mathcal{O}(\frac{1}{M})$

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We compute the $\mathcal{O}(\frac{1}{M}, m_s)$ heavy quark and SU(3) corrections to $\overline{B}_s \to D_s e\nu$ form factors. In the limit of vanishing light quark mass, $\overline{B}_s \to D_s e\nu$ form factors are given in terms of the the $\overline{B} \to De\nu$ form factors, the leading order chiral parameter g, and two $\mathcal{O}(\frac{1}{M})$ chiral parameters g_1 and g_2. All the chiral parameters can be extracted, in principle, from other heavy meson decays. Analytic counterterms proportional to the strange quark mass are presented for completeness, but no predictive power remains when they are included. Anomalously large loop corrections warn of poor convergence of the heavy quark chiral symmetry expansion for these processes. This suggests that naive extrapolations of $\overline{B} \to D$ form factors relying on heavy quark and chiral symmetries, as often used in monte carlo simulations of lattice QCD, may incur large errors.

February 1995

1. Introduction

In the standard model of electroweak interactions, the semileptonic decays $\overline{B}_q \to D_q e\overline{\nu}$ and $\overline{B}_q \to D_q^* e\overline{\nu}$ proceed via exchange of a charged vector boson and therefore the corresponding rates are given in terms of the form factors for the charged currents times the fundamental CKM parameter $|V_{cb}|$. The study of these form factors is of interest to those who may try to extract $|V_{cb}|$ from experimental measurements of the decay rates as well as to those who are interested in how well approximate chiral and heavy-quark symmetries work in nature.

The combination of chiral $SU(3)$ symmetry and heavy quark flavor-spin symmetries gives all 18 form factors of the charged current $J_\mu = \overline{c}\gamma_\mu(1-\gamma_5)b$ matrix elements between a B_q meson and a D_q or a D_q^* mesons, with $q = u, d, s$, in terms of a single 'Isgur-Wise' function. Because these are only approximate symmetries, the relations among form factors are not expected to hold exactly. SU(3) chiral log corrections to the relations between form factors have been examined at $\mathcal{O}(M^0)$[1]. $\mathcal{O}(\frac{1}{M^n})$ corrections involving hyperfine splitting and inverse powers of the pion mass have also been studied[2].

Here we present the first complete analysis of all $\mathcal{O}(\frac{1}{M})$, $SU(3)$ breaking corrections to the relations between $B_q \to \{D_q, D_q^*\}$ form factors. In addition to hyperfine corrections, we include the leading heavy quark symmetry breaking corrections in both the current and the lagrangian. We account both for terms that depend non-analytically on the symmetry breaking parameters $1/M$ and m_s, and are enhanced in the theoretical limit of small $1/M$ and m_s, and for terms with analytic dependence, which although suppressed in the chiral limit are often non-negligible in reality.

If the analytic counter-terms are indeed non-negligible, predictive power is lost. Of course, if they are small at some preordained renormalization point, then the dominant non-analytic terms can be calculated. We find that these non-analytic terms can be substantial. In fact, simultaneous violations of both chiral and heavy quark symmetries can be as large as 30%.

Determinations of form factors for $\overline{B} \to Dl\nu$ by Monte Carlo simulations of lattice QCD often extrapolate in heavy and light masses to the physical case[3]. Our study indicates that such extrapolations may incur large errors due to violations of heavy quark and chiral symmetries.

To investigate $B \to D$, we write a chiral lagrangian that describes the low energy interactions of single heavy mesons with pions. In this lagrangian all of the symmetries are

explicitly realized. We extend the lagrangian to include symmetry breaking terms to the order of interest, and retain only those terms that contribute to the $B \to \{D, D^*\}$ form factors. As we will see, these corrections involve heavy quark spin and flavor violating axial couplings g_1, g_2, which may be extracted, in principle, from heavy to light meson decays. An analogous analysis is done for the charged current operator, which is written in terms of the meson fields. One loop diagrams must be computed to complete the calculations, since they give terms of the same order in the expansion parameters $1/M$ and m_s (or sometimes even lower order, when they give non-analytic dependence on these parameters).

In section 2 we review the formulation of chiral lagrangians for interactions of heavy mesons with pions, and construct the lagrangian and charged current to linear order in $1/M$ and m_s. In the following section we describe the one loop calculation and present our results as corrections to hadronic form factors. We verify in section 4 that these results are consistent with Luke's theorem[4], then discuss physical implications in section 5. We find surprisingly large symmetry violations for a particular ratio of form factors, casting doubt on the convergence of the heavy quark-chiral expansion for these processes. We make concluding remarks in the final section.

2. Lagrangian and Current

The low momentum strong interactions of B and B^* (or D and D^*) mesons are governed by the chiral lagrangian [5]

$$\begin{aligned} \mathcal{L} = &- \mathrm{Tr}\left[\overline{H}_a(v) \mathrm{i} v\cdot D_{ba} H_b(v)\right] \\ &+ g\ \mathrm{Tr}\left[\overline{H}_a(v) H_b(v) \not{A}_{ba}\gamma_5\right]\ . \end{aligned} \tag{2.1}$$

Operators suppressed by powers of the heavy meson mass $1/M_B$, factors of a light quark mass m_q, or additional derivatives have been omitted. The field ξ contains the octet of pseudo-Nambu-Goldstone bosons

$$\xi = \exp\left(i\Pi/f\right), \tag{2.2}$$

where

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{2.3}$$

The bosons couple to heavy fields through the covariant derivative and the axial vector field,

$$
\begin{aligned}
D^\mu_{ab} &= \delta_{ab}\partial^\mu + V^\mu_{ab} = \delta_{ab}\partial^\mu + \tfrac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)_{ab} \,, \\
A^\mu_{ab} &= \frac{\mathrm{i}}{2}\left(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger\right)_{ab} = -\frac{1}{f}\partial_\mu\mathcal{M}_{ab} + \mathcal{O}(\mathcal{M}^3)\,.
\end{aligned} \tag{2.4}
$$

The B and B^* heavy meson fields are incorporated into the 4×4 matrix H_a:

$$
\begin{aligned}
H_a &= \frac{1}{2}\,(1+\not{v})\left[\overline{B}^{*\mu}_a\gamma_\mu - \overline{B}_a\gamma_5\right], \\
\overline{H}_a &= \gamma^0 H_a^\dagger\gamma^0\,.
\end{aligned} \tag{2.5}
$$

The four-velocity of the heavy meson is v^μ, and the index a runs over light quark flavor. The bar over B will sometimes be omitted for notational simplicity.

The lagrangian to $\mathcal{O}(\frac{1}{M}, m_s)$ may be written[6]

$$
\mathcal{L} = -\,\mathrm{Tr}\left[\overline{H}_a(v)\mathrm{i}v\cdot D_{ba}H_b(v)\right] + \tilde{g}_{\overline{H}H}\ \mathrm{Tr}\left[\overline{H}_a(v)H_b(v)\not{A}_{ba}\gamma_5\right] \tag{2.6}
$$

where

$$
\tilde{g} = \begin{cases} \tilde{g}_{B^*} = g + \frac{1}{M}(g_1+g_2) & \text{for } B^*B^* \text{ coupling,} \\ \tilde{g}_B = g + \frac{1}{M}(g_1-g_2) & \text{for } B^*B \text{ coupling.} \end{cases}
$$

Any $SU(3)$ counterterms relevant to $B\to Dl\overline{\nu}$ at this order may be accounted for by using normalized fields and physical masses in propagators. Our propagators are $\frac{\mathrm{i}}{2(v\cdot k+\frac{3}{4}\Delta)}$ for the B, $\frac{-\mathrm{i}(g^{\mu\nu}-v^\mu v^\nu)}{2(v\cdot k-\frac{1}{4}\Delta)}$ for the B^*, $\frac{\mathrm{i}}{2(v\cdot k-\delta+\frac{3}{4}\Delta)}$ for the B_s, and $\frac{-\mathrm{i}(g^{\mu\nu}-v^\mu v^\nu)}{2(v\cdot k-\delta-\frac{1}{4}\Delta)}$ for the B_s^*, where $\Delta = M_{B^*}-M_B$ is the hyperfine mass splitting and $\delta = M_{D_s}-M_D = M_{B_s}-M_B+\mathcal{O}(\frac{\overline{\Lambda}^2}{M})$ is the SU(3) mass splitting.

The current may be parametrized as

$$
\begin{aligned}
J^\lambda = &\left[-\xi_0(\omega)+\rho_1(\omega)(\frac{1}{M_D}+\frac{1}{M_B})\right]\mathrm{Tr}[\overline{H}(v)\Gamma H(v')] \\
&+\rho_2(\omega)\left[\frac{1}{M_D}\,\mathrm{Tr}[\gamma_5\overline{H}(v)\gamma_5\Gamma H(v')]+\frac{1}{M_B}\,\mathrm{Tr}[\gamma_5\overline{H}(v)\Gamma\gamma_5 H(v')]\right] \\
&+\rho_3(\omega)\left[\frac{1}{M_D}\,\mathrm{Tr}[\gamma_\mu\overline{H}(v)\gamma^\mu\Gamma H(v')]+\frac{1}{M_B}\,\mathrm{Tr}[\gamma_\mu\overline{H}(v)\Gamma\gamma^\mu H(v')]\right] \\
&+\rho_4(\omega)\left[\frac{1}{M_D}\,\mathrm{Tr}[\sigma_{\mu\nu}\overline{H}(v)\sigma^{\mu\nu}\Gamma H(v')]+\frac{1}{M_B}\,\mathrm{Tr}[\sigma_{\mu\nu}\overline{H}(v)\Gamma\sigma^{\mu\nu}H(v')]\right] \\
&+\rho_5(\omega)\left[\frac{1}{M_D}\,\mathrm{Tr}[\overline{H}(v)\not{v}'\Gamma H(v')]+\frac{1}{M_B}\,\mathrm{Tr}[\overline{H}(v)\Gamma\not{v}H(v')]\right] \\
&+\rho_6(\omega)\left[\frac{1}{M_D}\,\mathrm{Tr}[\gamma_\mu\overline{H}(v)\gamma^\mu\not{v}'\Gamma H(v')]+\frac{1}{M_B}\,\mathrm{Tr}[\gamma_\mu\overline{H}(v)\Gamma\not{v}\gamma^\mu H(v')]\right]
\end{aligned} \tag{2.7}
$$

where $\omega = v \cdot v'$ and $\Gamma = \gamma^\lambda(1-\gamma_5)$. Time reversal invariance dictates that all the ρ's be real.

If we take $M_B \to \infty$, and consider only $B_q \to \{D_q, D_q^*\}$ matrix elements, the resulting form for the current is valid to all orders in $\frac{1}{M_D}$. After all it involves seven independent functions, but the matrix elements can be expressed in terms of six form factors. However, at $\mathcal{O}(\frac{1}{M_D})$, heavy quark symmetry is broken in a specific fashion, and relations among the rho's exist. The simplest way to find these relations is to match onto the heavy quark effective theory[7], giving[4],

$$\begin{aligned}
\rho_1 &= -\xi_+ - \frac{\overline{\Lambda}}{2}\xi_0 - \chi_1 + \chi_2 - 2\chi_3 \\
\rho_2 &= 0 \\
\rho_3 &= -(1+\omega)\xi_+ - (1-\omega)\frac{\overline{\Lambda}}{2}\xi_0 + \omega\chi_2 - 2\chi_3 \\
\rho_4 &= -\chi_3 \\
\rho_5 &= -\xi_+ + \frac{\overline{\Lambda}}{2}\xi_0 + \chi_2 \\
\rho_6 &= \chi_2
\end{aligned} \tag{2.8}$$

In the notation of [4], ξ_0 is the leading order Isgur-Wise function, while ξ_+, χ_1, χ_2 and χ_3 are $\mathcal{O}(\frac{1}{M})$ corrections satisfying $\chi_1(\omega=1) = \chi_3(\omega=1) = 0$, and $\overline{\Lambda} = M_B - M_b$.

At $\mathcal{O}(\frac{1}{M^0}, m_s)$, the current contains $SU(3)$ violating terms proportional to the light quark mass matrix $m_q = \mathrm{diag}[0,0,m_s]$. To leading order in derivatives, the chiral symmetry breaking current is

$$\begin{aligned}
J^\lambda_{(m)} = -\xi_0\frac{\eta_0}{\Lambda_\chi}\,\mathrm{Tr}[\overline{H}(v)_b\Gamma H(v')_a]\mathcal{M}^+_{ab} + \frac{\kappa_1}{\Lambda_\chi}\,\mathrm{Tr}[\overline{H}(v)_a\Gamma H(v')_a]\mathcal{M}^+_{bb} \\
+ \frac{\kappa_2}{\Lambda_\chi}\,\mathrm{Tr}[\overline{H}(v)_b\Gamma H(v')_a\gamma_5]\mathcal{M}^-_{ab} + \frac{\kappa_3}{\Lambda_\chi}\,\mathrm{Tr}[\overline{H}(v)_a\Gamma H(v')_a\gamma_5]\mathcal{M}^-_{bb}
\end{aligned} \tag{2.9}$$

where $\mathcal{M}^\pm = \frac{1}{2}(\xi m_q \xi \pm \xi^\dagger m_q \xi^\dagger)_{ba}$.

To $\mathcal{O}(\frac{1}{M}, m_s)$, only operators linear in m_q and inserted in tree graphs are relevant, so we take $\xi \to 1$, $\mathcal{M}^+ \to m_q$, and $\mathcal{M}^- \to 0$. Terms with $\mathcal{M}^+_{aa}$ are $SU(3)$ symmetric and can be absorbed by redefinitions of the parameters ρ_1–ρ_6. This limits considerably the terms that need be considered: To $\mathcal{O}(M^0, m_s)$ only η_0 enters, while to $\mathcal{O}(\frac{1}{M}, m_s)$ we have

$$\begin{aligned}
J^\lambda_{(M+m)} = m^q_{ab}\Big\{(\frac{-\xi_0\eta_0}{\Lambda_\chi} + \frac{\rho_1\eta_1}{M_D})\,\mathrm{Tr}[\overline{H}_b(v)\Gamma H_a(v')] + \rho_2\frac{\eta_2}{M_D}\,\mathrm{Tr}[\gamma_5\overline{H}_b(v)\gamma_5\Gamma H_a(v')] \\
+ \rho_3\frac{\eta_3}{M_D}\,\mathrm{Tr}[\gamma_\mu\overline{H}_b(v)\gamma^\mu\Gamma H_a(v')] + \rho_4\frac{\eta_4}{M_D}\,\mathrm{Tr}[\sigma_{\mu\nu}\overline{H}_b(v)\sigma^{\mu\nu}\Gamma H_a(v')] \\
+ \rho_5\frac{\eta_5}{M_D}\,\mathrm{Tr}[\overline{H}_b(v)\not{v}'\Gamma H_a(v')] + \rho_6\frac{\eta_6}{M_D}\,\mathrm{Tr}[\gamma_\mu\overline{H}_b(v)\gamma^\mu\not{v}'\Gamma H_a(v')]\Big\}
\end{aligned} \tag{2.10}$$

Since we are ignoring up and down masses, the current in Eq. (2.10) only contributes to the $B_s \to \{D_s, D_s^*\}$ transitions. As in the $B \to \{D, D^*\}$ case, we can match onto the heavy quark effective theory. The η_2 term multiplies ρ_2, which vanishes. In addition, the parameters satisfy

$$-\rho_3\eta_3 + 2\rho_4\eta_4 + (1+\omega)\rho_5\eta_5 - \rho_6\eta_6 = \overline{\Lambda}_s\xi_0, \tag{2.11}$$

where $\overline{\Lambda}_s = M_{B_s} - m_b = \overline{\Lambda} + \delta$.

3. Calculation and Results

It is convenient to introduce form factors for $B^* \to \{D, D^*\}$ transitions in addition to those for $B \to \{D, D^*\}$. We define them as follows:

$$\begin{aligned}
\frac{1}{\sqrt{M_D M_{B^*}}}\langle D(v)|\,\overline{c}\gamma^\lambda(1-\gamma_5)b\,|B^*(\epsilon_B, v')\rangle &= f_0 v\cdot\epsilon_B v^\lambda + \mathrm{i} f_1 \epsilon^\lambda_{\alpha\beta\gamma}\epsilon_B^\alpha v^\beta v'^\gamma \\
&\quad + f_2 v\cdot\epsilon_B v'^\lambda + f_3\epsilon_B^\lambda \\
\frac{1}{\sqrt{M_D M_{B^*}}}\langle D^*(\epsilon_D, v)|\,\overline{c}\gamma^\lambda(1-\gamma_5)b\,|B^*(\epsilon_B, v')\rangle &= f_4\epsilon_B\cdot\epsilon_D^* v^\lambda + f_5\epsilon_B\cdot\epsilon_D^* v'^\lambda \\
&\quad + f_6 v\cdot\epsilon_B v'\cdot\epsilon_D^* v'^\lambda + f_7 v\cdot\epsilon_B v'\cdot\epsilon_D^* v^\lambda \\
&\quad + f_8 v'\cdot\epsilon_D^*\epsilon_B^\lambda + f_9 v\cdot\epsilon_B\epsilon_D^\lambda \\
&\quad + \mathrm{i} f_{10}\epsilon^\lambda_{\alpha\beta\gamma}\epsilon_B^\alpha\epsilon_D^{*\beta}v^\gamma + \mathrm{i} f_{11}\epsilon^\lambda_{\alpha\beta\gamma}\epsilon_B^\alpha\epsilon_D^{*\beta}v'^\gamma \\
&\quad + \mathrm{i} f_{12}(v\cdot\epsilon^\lambda_{\alpha\beta\gamma}\epsilon_B\epsilon_D^{*\alpha}v^\beta v'^\gamma + \epsilon_{\alpha\beta\gamma\delta}\epsilon_B^\alpha\epsilon_D{*}^\beta v^\gamma v'^\delta v^\lambda) \\
&\quad + \mathrm{i} f_{13}(v'\cdot\epsilon_D^*\epsilon^\lambda_{\alpha\beta\gamma}\epsilon_B^\alpha v^\beta v'^\gamma - \epsilon_{\alpha\beta\gamma\delta}\epsilon_B^\alpha\epsilon_D^{*\beta}v^\gamma v'^\delta v'^\lambda) \\
\frac{1}{\sqrt{M_D M_{B^*}}}\langle D^*(\epsilon_D, v)|\,\overline{c}\gamma^\lambda(1-\gamma_5)b\,|B(v')\rangle &= h_1 v'\cdot\epsilon_D^* v'^\lambda + h_2 v'\cdot\epsilon_D^* v^\lambda + h_3\epsilon_D^{*\lambda} \\
&\quad + \mathrm{i} h_4\epsilon^\lambda_{\alpha\beta\gamma}\epsilon_D^\alpha v^\beta v'^\gamma \\
\frac{1}{\sqrt{M_D M_{B^*}}}\langle D(v)|\,\overline{c}\gamma^\lambda(1-\gamma_5)b\,|B(v')\rangle &= h_5 v^\lambda + h_6 v'^\lambda
\end{aligned} \tag{3.1}$$

Other vector to vector form factors are simply related to f_{12} and f_{13} by the identity $\epsilon^{abcd}g^{\mu e} + \epsilon^{eabc}g^{\mu d} + \epsilon^{deab}g^{\mu c} + \epsilon^{cdea}g^{\mu b} + \epsilon^{bcde}g^{\mu a} = 0$. The Feynman rules for an insertion of the current are summarized in terms of these form factors in appendix A. We do the computation by using the form factors above in the one loop diagrams, replacing them by their values in terms of the chiral parameters ρ_i, g_i at the very end. We then express the results of the one loop computation as corrections to the form factors h_1 through h_6

(in the notation of [2], $h_1 = \tilde{a}_+ - \tilde{a}_-, h_2 = \tilde{a}_+ + \tilde{a}_-, h_3 = \tilde{f}, h_4 = \tilde{g}, h_5 = \tilde{f}_+ + \tilde{f}_-$, and $h_6 = \tilde{f}_+ - \tilde{f}_-$).

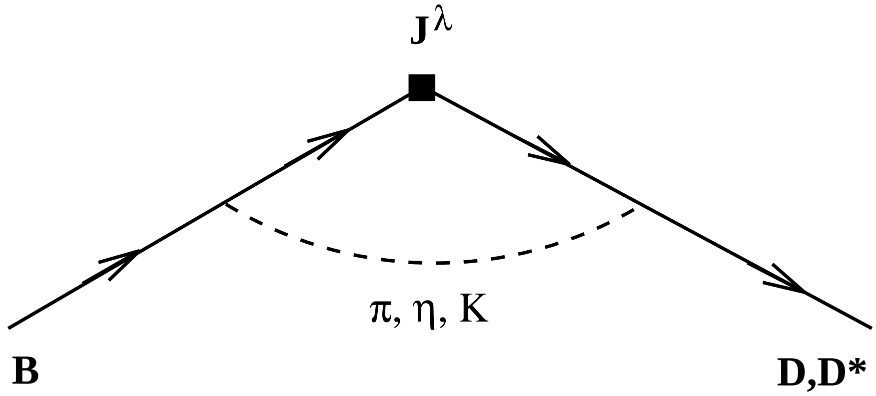

Figure 1. *Vertex correction graphs in the computation of chiral corrections to form factors for $B \to \{D, D^*\}$ form factors.*

We turn our attention to the one loop corrections. The only diagrams that need be computed are the vertex correction of fig. 1 and the self-energy graphs of fig. 2. The only integral needed is

$$\begin{aligned} C^{\alpha\beta}(\omega, m, \Delta, \Delta') &= \int \frac{\mathrm{d}^{4-\epsilon}\, k}{(2\pi)^{4-\epsilon}} \frac{k^\alpha k^\beta}{(k^2 - m^2)(v\cdot k - \Delta)(v'\cdot k - \Delta')} \\ &= \frac{\mathrm{i}}{16\pi^2}[C_1(\omega, m, \Delta, \Delta')g^{\alpha\beta} + C_2(\omega, m, \Delta, \Delta')(v^\alpha v'^\beta + v^\beta v'^\alpha)] \\ &+ C_3(\omega, m, \Delta, \Delta')v'^\alpha v'^\beta + C_4(\omega, m, \Delta, \Delta')v^\alpha v^\beta \end{aligned} \tag{3.2}$$

We express the four functions C_i as one dimensional integrals in Appendix B. Also in appendix B, we define and plot certain linear combinations, $C_i^{(j)}(\omega)$, of the functions $C_i(\omega)$, and linear combinations $D_i^{(j)}$ of the constants $C_i(1)$, that appear frequently in our results.

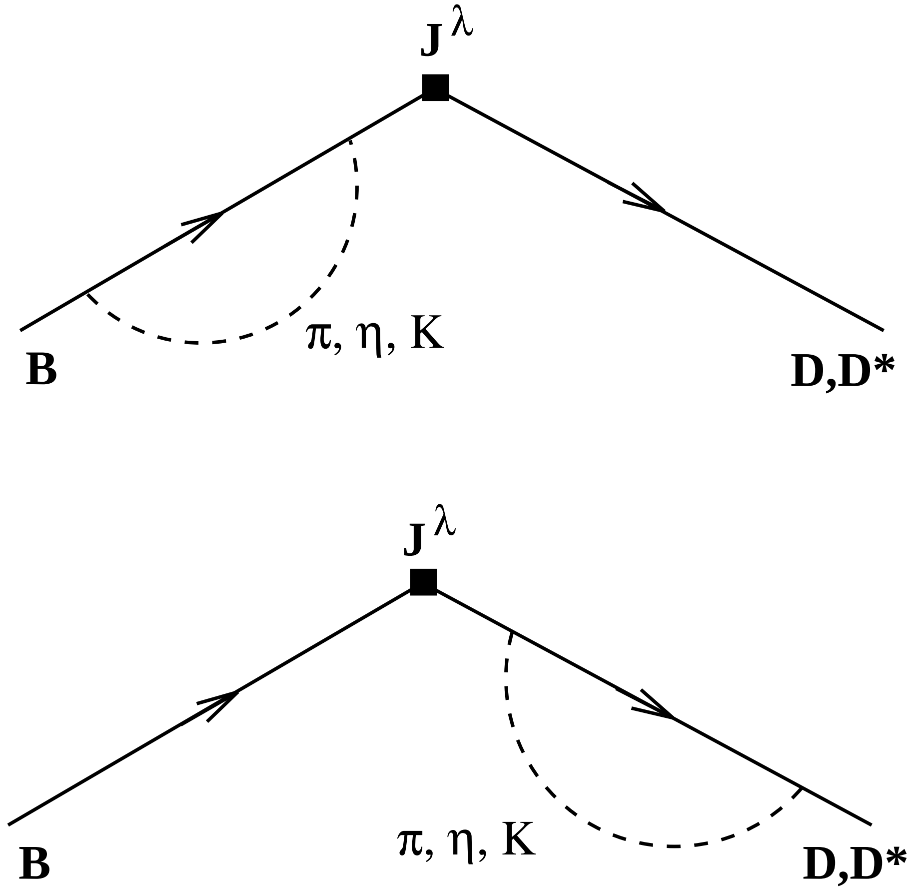

Figure 2. *Self-energy diagrams corntributing to the chiral corrections to form factors for $B \to \{D, D^*\}$.*

Wavefunction renormalization factors are given by

$$\begin{aligned} Z_B &= 1 - \frac{3g_B^2}{16\pi^2 f^2} D_1^{(1)} \\ Z_{B_s} &= 1 - \frac{3g_B^2}{16\pi^2 f^2} D_1^{(2)} \\ Z_{B^*} &= 1 - \frac{1}{16\pi^2 f^2}(2g_{B^*}^2 D_1^{(5)} + g_B^2 D_1^{(3)}) \\ Z_{B_s^*} &= 1 - \frac{1}{16\pi^2 f^2}(2g_{B^*}^2 D_1^{(6)} + g_B^2 D_1^{(4)}) \end{aligned} \tag{3.3}$$

Combining the results for vertex and self-energy graphs we find it now straightforward to obtain the form factors valid to $\mathcal{O}(\frac{1}{M}, m_s)$ at one loop. For decays to the pseudoscalar,

we have

$$
\begin{aligned}
h_5^{B\to D} = &\left(1-\frac{3(g_B^2 D_1^{(1B)}+g_D^2 D_1^{(1D)})}{32\pi^2 f^2}\right)\Big\{\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[(2-\omega)\overline{\Lambda}\xi_0+2(1+\omega)\xi_+ +\chi_1+2(1-\omega)\chi_2+6\chi_3\right] \\
&\qquad + \tfrac{1}{M_B}\left[-(2-\omega)\overline{\Lambda}\xi_0-2(1+\omega)\xi_+ +\chi_1+2(1-\omega)\chi_2+6\chi_3\right]\Big\} \\
&- \frac{g_B g_D}{16\pi^2 f^2}C_1^{(1)}\Big\{-(2+\omega)\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[-(2+\omega^2)\overline{\Lambda}\xi_0+2\omega(1+\omega)\xi_+ -(2+\omega)(\chi_1-2\chi_3)+2\omega(1-\omega)\chi_2\right] \\
&\qquad + \tfrac{1}{M_B}\left[(2+\omega^2)\overline{\Lambda}\xi_0-2\omega(1+\omega)\xi_+ -(2+\omega)(\chi_1-2\chi_3)+2\omega(1-\omega)\chi_2\right]\Big\} \\
&- \frac{g_B g_D}{16\pi^2 f^2}C_2^{(1)}(1-\omega^2)\Big\{\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[\omega\overline{\Lambda}\xi_0-2(1+\omega)\xi_+ +\chi_1-2(1-\omega)\chi_2-2\chi_3\right] \\
&\qquad + \tfrac{1}{M_B}\left[-\omega\overline{\Lambda}\xi_0+2(1+\omega)\xi_+ +\chi_1-2(1-\omega)\chi_2-2\chi_3\right]\Big\}
\end{aligned}
\tag{3.4}
$$

and

$$
\begin{aligned}
h_6^{B\to D} = &\left(1-\frac{3(g_B^2 D_1^{(1B)}+g_D^2 D_1^{(1D)})}{32\pi^2 f^2}\right)\Big\{\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[-(2-\omega)\overline{\Lambda}\xi_0-2(1+\omega)\xi_+ +\chi_1+2(1-\omega)\chi_2+6\chi_3\right] \\
&\qquad + \tfrac{1}{M_B}\left[(2-\omega)\overline{\Lambda}\xi_0+2(1+\omega)\xi_+ +\chi_1+2(1-\omega)\chi_2+6\chi_3\right]\Big\} \\
&\frac{g_B g_D}{16\pi^2 f^2}C_1^{(1)}\Big\{-(2+\omega)\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[(2+\omega^2)\overline{\Lambda}\xi_0-2\omega(1+\omega)\xi_+ -(2+\omega)(\chi_1-2\chi_3)+2\omega(1-\omega)\chi_2\right] \\
&\qquad + \tfrac{1}{M_B}\left[-(2+\omega^2)\overline{\Lambda}\xi_0+2\omega(1+\omega)\xi_+ -(2+\omega)(\chi_1-2\chi_3)+2\omega(1-\omega)\chi_2\right]\Big\} \\
&- \frac{g_B g_D}{16\pi^2 f^2}C_2^{(1)}(1-\omega^2)\Big\{\xi_0 \\
&\qquad + \tfrac{1}{M_D}\left[-\omega\overline{\Lambda}\xi_0+2(1+\omega)\xi_+ +\chi_1-2(1-\omega)\chi_2-2\chi_3\right] \\
&\qquad + \tfrac{1}{M_B}\left[\omega\overline{\Lambda}\xi_0-2(1+\omega)\xi_+ +\chi_1-2(1-\omega)\chi_2-2\chi_3\right]\Big\}
\end{aligned}
\tag{3.5}
$$

For decays to vectors, we have

$$
\begin{aligned}
h_4^{B\to D^*} = & \left(1-\frac{2g_{D^*}^2 D_1^{(5D)}+g_D^2 D_1^{(3D)}+3g_B^2 D_1^{(1B)}}{32\pi^2 f_K^2}\right)\Big\{-\xi_0+\tfrac{1}{M_D}\left[-\overline{\Lambda}\xi_0-\chi_1+2\chi_3\right] \\
& \quad +\tfrac{1}{M_B}\left[-(2-\omega)\overline{\Lambda}\xi_0-2(1+\omega)\xi_+-\chi_1-2(1-\omega)\chi_2-6\chi_3\right]\Big\} \\
& -\frac{g_B g_D}{16\pi^2 f^2}C_1^{(5)}\Big\{\xi_0+\tfrac{1}{M_D}\left[(2-\omega)\overline{\Lambda}\xi_0+2(1+\omega)\xi_++\chi_1+2(1-\omega)\chi_2+6\chi_3\right] \\
& \quad +\tfrac{1}{M_B}\left[\overline{\Lambda}\xi_0+\chi_1-2\chi_3\right]\Big\} \\
& -\frac{g_B g_{D^*}}{16\pi^2 f^2}C_1^{(3)}\Big\{(1+\omega)\xi_0 \\
& \quad +\tfrac{1}{M_D}\left[2\omega\overline{\Lambda}\xi_0-2(1+\omega)\xi_++(1+\omega)(\chi_1-2\chi_3)-2(1-\omega)\chi_2\right] \\
& \quad +\tfrac{1}{M_B}\left[(1+\omega^2)\overline{\Lambda}\xi_0-2\omega(1+\omega)\xi_++(1+\omega)(\chi_1-2\chi_3)-2\omega(1-\omega)\chi_2\right]\Big\} \\
& -\frac{g_B g_{D^*}}{16\pi^2 f^2}C_2^{(3)}(1-\omega^2)\Big\{\xi_0+\tfrac{1}{M_D}\left[\overline{\Lambda}\xi_0+\chi_1-2\chi_3\right] \\
& \quad +\tfrac{1}{M_B}\left[\omega\overline{\Lambda}\xi_0-2(1+\omega)\xi_++\chi_1-2(1-\omega)\chi_2-2\chi_3\right]\Big\}
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
h_3^{B\to D^*} = & \left(1-\frac{2g_{D^*}^2 D_1^{(5D)}+g_D^2 D_1^{(3D)}+3g_B^2 D_1^{(1B)}}{32\pi^2 f_K^2}\right)\Big\{-(1+\omega)\xi_0 \\
& \quad +\tfrac{1}{M_D}\left[(1-\omega)\overline{\Lambda}\xi_0-(1+\omega)(\chi_1-2\chi_3)\right] \\
& \quad +\tfrac{1}{M_B}\left[-(2-\omega)(1-\omega)\overline{\Lambda}\xi_0-2(1-\omega^2)(\xi_++\chi_2)-(1+\omega)(\chi_1+6\chi_3)\right]\Big\} \\
& -\frac{g_B g_D}{16\pi^2 f^2}C_1^{(5)}\Big\{(1+\omega)\xi_0 \\
& \quad +\tfrac{1}{M_D}\left[-(2-\omega)(1-\omega)\overline{\Lambda}\xi_0-2(1-\omega^2)(\xi_+-\chi_2)+(1+\omega)(\chi_1+6\chi_3)\right] \\
& \quad +\tfrac{1}{M_B}\left[-(1-\omega)\overline{\Lambda}\xi_0+(1+\omega)(\chi_1-2\chi_3)\right]\Big\} \\
& -\frac{g_B g_{D^*}}{16\pi^2 f^2}C_1^{(3)}\Big\{(1+\omega)^2\xi_0 \\
& \quad +\tfrac{1}{M_D}\left[-2\omega(1-\omega)\overline{\Lambda}\xi_0+2(1-\omega^2)(\xi_+-\chi_2)+(1+\omega)^2(\chi_1-2\chi_3)\right] \\
& +\tfrac{1}{M_B}\left[-(1-\omega)(1+\omega^2)\overline{\Lambda}\xi_0+2\omega(1-\omega^2)(\xi_+-\chi_2)+(1+\omega)^2(\chi_1-2\chi_3)\right]\Big\} \\
& -\frac{g_B g_{D^*}}{16\pi^2 f^2}(1-\omega^2)C_2^{(3)}\Big\{-(1+\omega)\xi_0+\tfrac{1}{M_D}\left[(1-\omega)\overline{\Lambda}\xi_0-(1+\omega)(\chi_1-2\chi_3)\right] \\
& \quad +\tfrac{1}{M_B}\left[\omega(1-\omega)\overline{\Lambda}\xi_0-2(1-\omega^2)(\xi_+-\chi_2)-(1+\omega)(\chi_1-2\chi_3)\right]\Big\}
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
h_2^{B\to D^*} = &\left(1-\frac{2g_{D^*}^2D_1^{(5D)}+g_D^2D_1^{(3D)}+3g_B^2D_1^{(1B)}}{32\pi^2f_K^2}\right)\Big\{\xi_0+\tfrac{1}{M_D}\left[2\xi_++\chi_1-2\chi_2-2\chi_3\right] \\
&\qquad+\tfrac{1}{M_B}\left[(2-\omega)\overline{\Lambda}\xi_0+2(1+\omega)\xi_++\chi_1+2(1-\omega)\chi_2+6\chi_3\right]\Big\} \\
&-\frac{g_Bg_D}{16\pi^2f^2}\omega C_1^{(5)}\Big\{\tfrac{1}{M_B}\left[\overline{\Lambda}\xi_0-2\xi_+-2\chi_2\right]\Big\} \\
&-\frac{g_Bg_D}{16\pi^2f^2}C_2^{(5)}\Big\{(1+\omega)\xi_0 \\
&\qquad+\tfrac{1}{M_D}\left[-(2-\omega)(1-\omega)\overline{\Lambda}\xi_0-2(1-\omega)^2(\xi_+-\chi_2)+(1+\omega)(\chi_1+6\chi_3)\right] \\
&\qquad+\tfrac{1}{M_B}\left[-(1-\omega)\overline{\Lambda}\xi_0+(1+\omega)(\chi_1-2\chi_3)\right]\Big\} \\
&-\frac{g_Bg_{D^*}}{16\pi^2f^2}C_1^{(3)}\Big\{-(2+\omega)\xi_0 \\
&\qquad+\tfrac{1}{M_D}\left[2(1-\omega)\overline{\Lambda}\xi_0+2\omega(\xi_+-\chi_2)-(2+\omega)(\chi_1-2\chi_3)\right] \\
&\qquad+\tfrac{1}{M_B}\left[-(2-\omega+\omega^2)\overline{\Lambda}\xi_0+2\omega^2(\xi_+-\chi_2)-(2+\omega)(\chi_1-2\chi_3)\right]\Big\} \\
&-\frac{g_Bg_{D^*}}{16\pi^2f^2}C_2^{(3)}\Big\{-\omega(1+\omega)\xi_0+\tfrac{1}{M_D}\left[\omega(1-\omega)\overline{\Lambda}\xi_0-\omega(1+\omega)(\chi_1-2\chi_3)\right] \\
&\qquad+\tfrac{1}{M_B}[-\tfrac{1}{2}(1+\omega-3\omega^2+\omega^3)\overline{\Lambda}\xi_0 \\
&\qquad\qquad+(1-\omega)(1-\omega^2)(\xi_+-\chi_2)-\omega(1+\omega)(\chi_1-2\chi_3)]\Big\}
\end{aligned}
\tag{3.8}
$$

$$
\begin{aligned}
h_1^{B\to D^*} &= \left(1 - \frac{2g_{D^*}^2 D_1^{(5D)} + g_D^2 D_1^{(3D)} + 3g_B^2 D_1^{(1B)}}{32\pi^2 f_K^2}\right)\left\{\tfrac{1}{M_D}\left[-\overline{\Lambda}\xi_0 + 2\xi_+ + 2\chi_2\right]\right\} \\
&\quad - \frac{g_B g_D}{16\pi^2 f^2}\omega C_1^{(5)}\Big\{-\xi_0 + \tfrac{1}{M_D}\left[(2-\omega)\overline{\Lambda}\xi_0 + 2(1+\omega)\xi_+ - \chi_1 - 2(1-\omega)\chi_2 - 6\chi_3\right] \\
&\qquad\qquad + \tfrac{1}{M_B}\left[(1-\omega)\overline{\Lambda}\xi_0 + 2\omega\xi_+ - \chi_1 - 2\omega\chi_2 + 2\chi_3\right]\Big\} \\
&\quad - \frac{g_B g_D}{16\pi^2 f^2} C_2^{(5)}\Big\{-(1+\omega)\xi_0 \\
&\qquad\qquad + \tfrac{1}{M_D}\left[-(2-3\omega+\omega^2)\overline{\Lambda}\xi_0 - 2(1-\omega^2)(\xi_+ + \chi_2) - (1+\omega)(\chi_1 + 6\chi_3)\right] \\
&\qquad\qquad + \tfrac{1}{M_B}\left[\omega(1-\omega)\overline{\Lambda}\xi_0 - 2(1-\omega^2)(\xi_+ - \chi_2) - (1+\omega)(\chi_1 - 2\chi_3)\right]\Big\} \\
&\quad - \frac{g_B g_{D^*}}{16\pi^2 f^2} C_1^{(3)}\Big\{\xi_0 + \tfrac{1}{M_D}\left[2\xi_+ + \chi_1 - 2\chi_2 - 2\chi_3\right] \\
&\qquad\qquad + \tfrac{1}{M_B}\left[-(1-\omega)\overline{\Lambda}\xi_0 - 2\omega\xi_+ + \chi_1 + 2\omega\chi_2 - 2\chi_3\right]\Big\} \\
&\quad - \frac{g_B g_{D^*}}{16\pi^2 f^2} C_2^{(3)}\Big\{(1+\omega)\xi_0 + \tfrac{1}{M_D}\left[-(1-\omega)\overline{\Lambda}\xi_0 + (1+\omega)(\chi_1 - 2\chi_3)\right] \\
&\qquad\qquad + \tfrac{1}{M_B}\left[-\omega(1-\omega)\overline{\Lambda}\xi_0 + 2(1-\omega^2)(\xi_+ - \chi_2) + (1+\omega)(\chi_1 - 2\chi_3)\right]\Big\}
\end{aligned} \tag{3.9}
$$

Eqs. (3.4)–(3.9) constitute our main result. They express the form factors for $B \to \{D, D^*\}$ in terms of five non-perturbative form factors and the computable functions listed in Appendix B. The corresponding form factors for $B_s \to \{D_s, D_s^*\}$, $h_i^{B_s\to D_s}$, may be obtained from those above by the substitutions $C_i^{(m)} \to C_i^{(m+1)}$ and $D_i^{(m)} \to D_i^{(m+1)}$. Our form factors agree with reference [1] in the $M \to \infty$ limit.

No analytic counterterms have been included in these formulas. As one can see from eq. (2.10), including analytic counterterms is equivalent to defining a strange system Isgur-Wise function $\xi_0^{(s)}$ and new $\mathcal{O}(\frac{1}{M})$ correction functions $\xi_-^{(s)}$, $\xi_+^{(s)}$, $\chi_1^{(s)}$, $\chi_2^{(s)}$, $\chi_3^{(s)}$. Thus, predictive power is lost. The non-analytic corrections remain interesting however, because many phenomenological models omit or improperly account for them (for example, non-relativistic quark models or quenched lattice calculations[8]), and may be improved by making them consistent with the above formulas. Along these lines, an examination of the effect of quenching on heavy meson decay constants has been performed in reference [9]. The present work allows a similar analysis to be done for semileptonic $B \to D$. The theoretical relation between strange and non-strange form factors is especially important for lattice computations, which typically extrapolate non-strange form factors from simulations involving strange quarks[3].

4. Luke's Theorem

We may check our calculation by verifying consistency with Luke's Theorem, which says $\frac{1}{M}$ corrections to our hadronic matrix elements must vanish at threshold. It is easy to see that this holds true for $\langle D(v)|\,\overline{c}\gamma^{\lambda}(1-\gamma_5)b\,|B(v)\rangle$, but the decay to D^* is less transparent.

The subleading Isgur-Wise functions drop out of the only surviving $B \to D^*$ form factor h_3 at threshold because the coefficients of $\overline{\Lambda}\xi_0, \xi_+$, and χ_2 are proportional to $1-\omega$, while the functions χ_1 and χ_3 vanish identically at $\omega = 1$. The sum of $\overline{B} \to D^*$ graphs is thus

$$\begin{aligned}
&\langle D^*(\epsilon_D, v)|\,\overline{c}\gamma^{\lambda}(1-\gamma_5)b\,|B(v)\rangle = -2\epsilon_D^{*\lambda} \\
&- \epsilon_D^{*\lambda}\frac{1}{16\pi^2 f_K^2}\left[2g_B g_D C_1^{(5)}\Big|_{\omega=1} + 4g_B g_{D^*} C_1^{(3)}\Big|_{\omega=1} - 2g_{D^*}^2 D_1^{(5D)} - g_D^2 D_1^{(3D)} - 3g_B^2 D_1^{(1B)}\right]
\end{aligned} \tag{4.1}$$

Luke's theorem requires that the term in brackets vanish to $\mathcal{O}(\frac{1}{M})$. The $\mathcal{O}(1)$ terms vanish trivially because the various integrals $C_1^{(i)}$, $D_1^{(i)}$ are equal to each other at $\omega = 1$ to $\mathcal{O}(\frac{1}{M})$. There are two contributions at $\mathcal{O}(\frac{1}{M}, m_s)$. The $\mathcal{O}(\frac{1}{M})$ pieces of the axial couplings can multiply the $\mathcal{O}(m_s)$ parts of the integrals to give a sum proportional to

$$2g_B g_D + 4g_B g_{D^*} - 2g_{D^*}^2 - g_D^2 - 3g_B^2 = 0 + \mathcal{O}(\frac{1}{M_D^2}, \frac{1}{M_B^2}, \frac{1}{M_B M_D}),$$

or the $\mathcal{O}(\frac{1}{M}, m_s)$ parts of the integrals can multiply the $\mathcal{O}(1)$ axial couplings to give a sum proportional to

$$2C_1^{(5)} + 4C_1^{(3)} - 2D_1^{(5D)} - D_1^{(3D)} - 3D_1^{(1D)}.$$

By writing $a = \frac{\delta}{m}(\cos\Phi + \sin\Phi) + x$ and expanding the integrals to linear order in x, one can show that this sum is proportional to

$$\int d\Phi K(b, \delta(\cos\Phi + \sin\Phi))(3\Delta^B + \Delta^D)(\cos\Phi - \sin\Phi) = 0,$$

where K is a function even under interchange of $\cos\Phi$ and $\sin\Phi$.

5. Discusion

Previous analyses[2] included only $\frac{1}{M}$ corrections due to D meson hyperfine splitting. Here, we also include B meson hyperfine splittings, $\mathcal{O}(\frac{1}{M})$ axial coupling corrections, and $\mathcal{O}(\frac{1}{M})$ corrections to the current.

We identify contributions which are readily separated from contact terms by their parametric behavior in the $m \to 0$ and $\Delta \to 0$ limits. This includes both chiral logs which go like $m^2 \ln \frac{\mu^2}{m^2}$ and functions which depend on the ratio $\frac{\Delta}{m}$. Such behavior can never arise from contact terms because the counterterms are proportional to positive, integer powers of the light quark masses.

The nonanalytic dependence of the form factors in eqs. (3.4) to (3.9) arises from the integrals C_i in eqs. (B.2) and (B.3) of Appendix B. These integrals are finite in the $m \to 0$ and $\Delta, \Delta' \to 0$ limits, and may be expanded about $m = 0$ or $\Delta = 0$ after choosing the behavior of the ratio $\frac{\Delta}{m}$ in the double limit. At a fixed order in $\frac{1}{M}$, terms in C_i which go like $(\frac{\Delta}{m})^n$ dominate the heavy quark chiral expansion[2] if Δ is held fixed as $m \to 0$ (terms which go like $\Delta^2 \ln m^2$ cancel). However, if one does not assume this behavior of $\frac{\Delta}{m}$ (for example, one might instead hold the ratio fixed at a physical value), other contributions in the heavy quark chiral expansion (such as axial corrections g_i and current corrections ρ_i) are equally or more important.

Because the functions C_i are proportional to at least two powers of Δ or Δ', the $\mathcal{O}(\frac{1}{M}, m_s)$ corrections in eqs. (3.4) to (3.9) are proportional to either axial couplings g_1, g_2 or subleading Isgur-Wise functions $\overline{\Lambda}\xi_0, \xi_+, \chi_1, \chi_2, \chi_3$. It is only at $\mathcal{O}(\frac{1}{M^2})$ or $\mathcal{O}(\frac{1}{M}, m_s^2)$ that hyperfine splittings also enter.

In principle, our results can be used to estimate $B_s \to D_s l\nu$ form factors once $B \to Dl\nu$ form factors are measured. The necessary chiral lagrangian parameters g, g_1 and g_2 can be extracted from processes such as $B \to \pi l\nu$ and $B \to l\nu$[6], $D^* \to D\pi$[5], and $D^* \to D\gamma$[10]. For a more reliable estimate, analytic counterterms should be included (from a model or lattice computation).

To see the effect of the $\mathcal{O}(\frac{1}{M})$ and SU(3) corrections, we examine a quantity which is sensitive only to simultaneous violations of both symmetries, the ratio $R(\omega) = \frac{h_5^{B_s\to D_s}/h_6^{B_s\to D_s}}{h_5^{B\to D}/h_6^{B\to D}}$. In general, such ratios of form factors will depend on all the chiral parameters and subleading Isgur-Wise functions, but because of constraints due to Luke's

theorem, this particular ratio takes a simple form,

$$R(\omega)-1=\frac{-g^2}{16\pi^2 f^2}\left(\frac{\overline{\Lambda}}{M_D}-\frac{\overline{\Lambda}}{M_B}\right)\Bigg\{\Big[3(2-\omega)\left(D_1^{(2B)}-D_1^{(1B)}+D_1^{(2D)}-D_1^{(1D)}\right)$$
$$-2(2+\omega^2)\left(C_1^{(2)}-C_1^{(1)}\right)+2\omega(1-\omega^2)\left(C_2^{(2)}-C_2^{(1)}\right)\Big]$$
$$+\frac{2\xi_+}{\overline{\Lambda}\xi_0}(1+\omega)\Big[3\left(D_1^{(2B)}-D_1^{(1B)}+D_1^{(2D)}-D_1^{(1D)}\right)$$
$$+2\omega\left(C_1^{(2)}-C_1^{(1)}\right)-2(1-\omega^2)\left(C_2^{(2)}-C_2^{(1)}\right)\Big]\Bigg\}. \tag{5.1}$$

We retain the full dependence on the ratio $\frac{\Delta}{m}$, but dependence on g_1, g_2, χ_1, χ_2 and χ_3 drop out, to leading order in $\frac{1}{M}$.

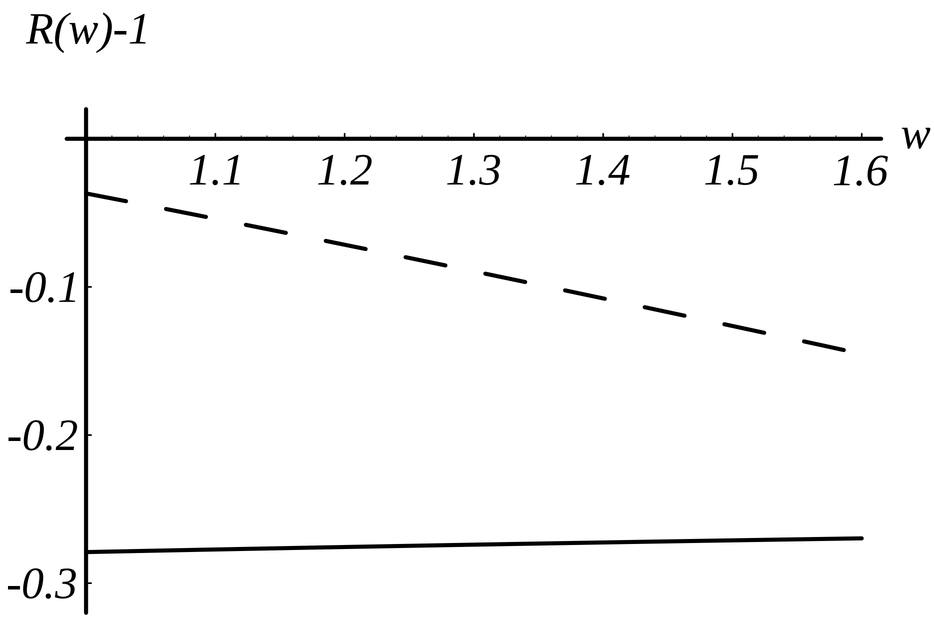

Figure 3. *The quantity* $R(\omega)-1 \equiv \frac{h_5^{B_s\to D_s}/h_6^{B_s\to D_s}}{h_5^{B\to D}/h_6^{B\to D}}-1$ *as given by eqn. (5.1), with* $g^2=0.5$, $\mu=1.0$ GeV *and* $\overline{\Lambda}=0.5$ GeV. *For the solid line we used* $(1+\omega)\frac{2\xi_+}{\overline{\Lambda}\xi_0}=\omega-2.2\pm0.4$ *from a QCD sum rule computation[11]. For the dashed line we used* $\xi_+=0$.

For a rough idea of the size and shape of $R(\omega)$, we take $g^2 = 0.5$, $\mu = 1.0$ GeV, and use the results of a QCD sum rule computation[11] as input: $\overline{\Lambda} = 0.5$ GeV and $(1+\omega)\frac{2\xi_+}{\overline{\Lambda}\xi_0} = \omega - 2.2 \pm 0.4$. The solid line in fig. 3 shows $R(\omega) - 1$ for these values while the dashed line shows it for $\xi_+ = 0$.

The surprisingly large deviation from unity arises because every term inside the parentheses of eq. (5.1) adds constructively. Special values of $\frac{\xi_+}{\overline{\Lambda}\xi_0}$ may give smaller deviations from unity, but fig. 3 represents the typical scale of symmetry breaking for this quantity. In order to estimate the size of the counterterms we study $R(\omega) - 1$ as we increase μ. For $\mu = 2$ GeV the dashed curve in fig. 3 is virtually unchanged but the solid curve nearly doubles. Such a large effect casts doubt on the validity of the simultaneous heavy quark and SU(3) chiral symmetry expansion for this system. Even with $\overline{\Lambda}$ as small as 0.25 GeV, the roughly 15% corrections to $R(\omega) - 1$ are much larger than expected. Thus, care should be taken when extrapolating form factors from the strange to non-strange $B \to Dl\nu$ systems, especially if, as in lattice calculations[3], both heavy quark and chiral extrapolations are performed.

6. Conclusions

We have computed the $\mathcal{O}(\frac{1}{M}, m_s)$ heavy quark and SU(3) corrections to $B_s \to D_s e\nu$ form factors. If the analytic counterterms are neglected, $B_s \to D_s e\nu$ form factors are given in terms of the the $B \to De\nu$ form factors, the leading order chiral parameter g, and two $\mathcal{O}(\frac{1}{M})$ chiral parameters g_1 and g_2. All the chiral parameters can be extracted, in principle, from other heavy meson decays, so all six potentially observable $B_s \to D_s l\nu$ form factors are determined, in the formal chiral limit, by the six form factors of $B \to Dl\nu$.

We say "formal" because, while the non-analytic terms are the leading terms in the chiral expansion for small kaon mass, in reality one expects analytic counterterms to be numerically just as important. Analytic counterterms proportional to the strange quark mass have also been presented, but no predictive power remains when they are included. The chiral log corrections remain interesting however, because many phenomenological models (such as nonrelativistic quark models or quenched lattice calculations) omit or improperly account for the chiral log contribution and may be improved by making them consistent with the above formulas. In addition, when such loop corrections are large, they warn of a possible breakdown of the chiral expansion.

Precisely this situation occurs for the quantity $R(\omega) = \frac{h_5^{B_s \to D_s}/h_6^{B_s \to D_s}}{h_5^{B \to D}/h_6^{B \to D}}$, where we find typical deviations from the symmetry limit of 15% to 30%. This is alarming, since these deviations involve simultaneous heavy quark and chiral symmetry violations, and are therefore expected to be only a few percent. Care should be taken when using heavy quark and SU(3) symmetry relations for the extraction of $B \to Dl\nu$ form factors from those of $B_s \to D_s l\nu$.

ACKNOWLEDGMENTS

B.G. is supported in part by the Alfred P. Sloan Foundation. G.B. and B.G. acknowledge the support of the Department of Energy, under contract DOE–FG03–90ER40546.

Appendix A. Feynman Rules for the Current

The Feynman rules for the current may be conveniently summarized by expressing the form factors in terms of the current parameters. Tree level, $\mathcal{O}(\frac{1}{M_D}, m_s^0)$ values for these form factors are given in table 1 (for example, $f_1 = \xi_0 + \frac{1}{M_D}[\rho_1 + 2\rho_3 - \rho_5 - 2\rho_6] + \mathcal{O}(\frac{1}{M_B})$).

Table 1.

	ρ_1	ρ_3	ρ_4	ρ_5	ρ_6
f_0	0	0	0	0	0
f_1	1	2	0	-1	-2
f_2	-1	-2	0	1	2
f_3	$1+\omega$	$-4+2\omega$	12	$1+\omega$	$-4+2\omega$
f_4	1	0	-4	-1	0
f_5	1	-2	0	$1+2\omega$	-2
f_6	0	0	0	-2	0
f_7	0	0	0	0	0
f_8	-1	2	0	1	-2
f_9	-1	0	4	1	0
f_{10}	-1	0	4	-1	0
f_{11}	-1	2	0	$-1-\omega$	2
f_{12}	0	0	0	0	0
f_{13}	0	0	0	1	0
h_1	0	0	0	-2	4
h_2	-1	0	4	-1	0
h_3	$1+\omega$	-2	-4ω	$1+\omega$	-2
h_4	1	0	-4	-1	0
h_5	-1	-2	0	1	2
h_6	-1	4	-12	$-1-2\omega$	$4-4\omega$

Heavy quark symmetry gives the $\mathcal{O}(\frac{1}{M_B})$ parts f_i^B, h_i^B, of these form factors simply in terms of the $\mathcal{O}(\frac{1}{M_D})$ parts f_i^D, h_i^D, by the following relations:

$$\begin{array}{lllll} f_0^B = -h_1^D & f_1^B = h_4^D & f_2^B = h_2^D & f_3^B = h_3^D & f_4^B = h_5^D \\ f_5^B = f_4^D & f_6^B = 0 & f_7^B = f_6^D & f_8^B = f_9^D & f_9^B = f_8^D \\ f_{10}^B = f_{11}^D & f_{11}^B = f_{10}^D & f_{12}^B = f_{13}^D & f_{13}^B = f_{12}^D & h_1^B = 0 \\ h_2^B = h_5^D & h_3^B = f_3^D & h_4^B = f_1^D & h_5^B = h_6^D & h_6^B = h_5^D \end{array} \tag{A.1}$$

Appendix B. Integral and definitions

The one loop corrections involve the integral

$$\begin{aligned} C^{\alpha\beta}(\omega, m, \Delta, \Delta') &= \int \frac{\mathrm{d}^{4-\epsilon}\, k}{(2\pi)^{4-\epsilon}} \frac{k^\alpha k^\beta}{(k^2 - m^2)(v \cdot k - \Delta)(v' \cdot k - \Delta')} \\ &= \frac{\mathrm{i}}{16\pi^2}[C_1(\omega, m, \Delta, \Delta') g^{\alpha\beta} + C_2(\omega, m, \Delta, \Delta')(v^\alpha v'^\beta + v^\beta v'^\alpha)] \\ &+ C_3(\omega, m, \Delta, \Delta') v'^\alpha v'^\beta + C_4(\omega, m, \Delta, \Delta') v^\alpha v^\beta \end{aligned} \tag{B.1}$$

The integrals $C_1 - C_4$ may be expressed as one dimensional integrals

$$\begin{aligned} C_1 = m^2 \int_0^{\frac{\pi}{2}} d\Phi \Big[& (\frac{2}{\epsilon} + \ln 4\pi - \gamma_E + 1)(2a^2b^2 - b) + 2a^2b^2 \\ & + (b - 2a^2b^2) \ln \frac{m^2}{\mu^2} - 4ab\sqrt{a^2b^2 - b}\, \ln[a\sqrt{b} + \sqrt{a^2b - 1}] \Big] \end{aligned} \tag{B.2}$$

$$\begin{aligned} C_i = -2m^2 \int_0^{\frac{\pi}{2}} d\Phi\, (\alpha_i(\Phi)) \Big[& -b^2(1 - 4a^2b)(\frac{2}{\epsilon} + \ln 4\pi - \gamma_E + 1) \\ & + b^2(1 - 4a^2b) \ln \frac{m^2}{\mu^2} + b^2(1 - 5a^2b) + \frac{2ab^3(3 - 4a^2b)}{\sqrt{a^2b^2 - b}} \ln[a\sqrt{b} + \sqrt{a^2b - 1}] \Big] \end{aligned} \tag{B.3}$$

where $a = \frac{\Delta}{m}\cos\Phi + \frac{\Delta'}{m}\sin\Phi$, $b = (1 + 2\omega\cos\Phi\sin\Phi)^{-1}$, and $\alpha_2(\Phi) = \cos\Phi\sin\Phi$, $\alpha_3(\Phi) = \sin^2\Phi$, $\alpha_4(\Phi) = \cos^2\Phi$.

We will work in a scheme such that $\frac{2}{\epsilon} + \ln 4\pi - \gamma_E + 1 = 0$. In this scheme, for $\Delta = \Delta' = 0$, these integrals simplify to

$$\begin{aligned} C_1 &= m^2 \ln \frac{m^2}{\mu^2} r(\omega), \\ C_2 &= -m^2 \frac{(\ln \frac{m^2}{\mu^2} + 1)}{1 - \omega^2}[1 - \omega r(\omega)], \\ C_3 = C_4 &= -m^2 \frac{(\ln \frac{m^2}{\mu^2} + 1)}{1 - \omega^2}[r(\omega) - \omega] \end{aligned} \tag{B.4}$$

where $r(\omega) = \frac{\ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}}$ and $r(\omega) \to 1$ at threshold.

The following linear combinations of the integrals C_i will be useful:

$$
\begin{aligned}
C_i^{(1)} &= C_i(\omega, m_K, \Delta^{(B)} + \delta, \Delta^{(D)} + \delta) \\
&\qquad + \tfrac{3}{2} C_i(\omega, m_\pi, \Delta^{(B)}, \Delta^{(D)}) + \tfrac{1}{6} C_i(\omega, m_\eta, \Delta^{(B)}, \Delta^{(D)}) \\
C_i^{(2)} &= 2C_i(\omega, m_K, \Delta^{(B)} - \delta, \Delta^{(D)} - \delta) + \tfrac{2}{3} C_i(\omega, m_\eta, \Delta^{(B)}, \Delta^{(D)}) \\
C_i^{(3)} &= C_i(\omega, m_K, \Delta^{(B)} + \delta, \delta) + \tfrac{3}{2} C_i(\omega, m_\pi, \Delta^{(B)}, 0) + \tfrac{1}{6} C_i(\omega, m_\eta, \Delta^{(B)}, 0) \\
C_i^{(4)} &= 2C_i(\omega, m_K, \Delta^{(B)} - \delta, -\delta) + \tfrac{2}{3} C_i(\omega, m_\eta, \Delta^{(B)}, 0) \\
C_i^{(5)} &= C_i(\omega, m_K, \Delta^{(B)} + \delta, -\Delta^{(D)} + \delta) \\
&\qquad + \tfrac{3}{2} C_i(\omega, m_\pi, \Delta^{(B)}, -\Delta^{(D)}) + \tfrac{1}{6} C_i(\omega, m_\eta, \Delta^{(B)}, -\Delta^{(D)}) \\
C_i^{(6)} &= 2C_i(\omega, m_K, \Delta^{(B)} - \delta, -\Delta^{(D)} - \delta) + \tfrac{2}{3} C_i(\omega, m_\eta, \Delta^{(B)}, -\Delta^{(D)})
\end{aligned}
\tag{B.5}
$$

are produced when summing over intermediate states contributing to vertex corrections, while

$$
\begin{aligned}
D_i^{(1B)} &= C_i(1, m_K, \Delta^{(B)} + \delta, \Delta^{(B)} + \delta) \\
&\qquad + \tfrac{3}{2} C_i(1, m_\pi, \Delta^{(B)}, \Delta^{(B)}) + \tfrac{1}{6} C_i(1, m_\eta, \Delta^{(B)}, \Delta^{(B)}) \\
D_i^{(2B)} &= 2C_i(1, m_K, \Delta^{(B)} - \delta, \Delta^{(B)} - \delta) + \tfrac{2}{3} C_i(1, m_\eta, \Delta^{(B)}, \Delta^{(B)}) \\
D_i^{(3B)} &= C_i(1, m_K, -\Delta^{(B)} + \delta, -\Delta^{(B)} + \delta) \\
&\qquad + \tfrac{3}{2} C_i(1, m_\pi, -\Delta^{(B)}, -\Delta^{(B)}) + \tfrac{1}{6} C_i(1, m_\eta, -\Delta^{(B)}, -\Delta^{(B)}) \\
D_i^{(4B)} &= 2C_i(1, m_K, -\Delta^{(B)} - \delta, -\Delta^{(B)} - \delta) + \tfrac{2}{3} C_i(1, m_\eta, -\Delta^{(B)}, -\Delta^{(B)}) \\
D_i^{(5B)} &= C_i(1, m_K, \delta, \delta) + \tfrac{3}{2} C_i(1, m_\pi, 0, 0) + \tfrac{1}{6} C_i(1, m_\eta, 0, 0) \\
D_i^{(6B)} &= 2C_i(1, m_K, -\delta, -\delta) + \tfrac{2}{3} C_i(\omega, m_\eta, 0, 0)
\end{aligned}
\tag{B.6}
$$

and the analogous integrals, $D_i^{(jD)}$ with $\Delta^{(B)} \to \Delta^{(D)}$, arise from wavefunction renormalization.

The real parts of the functions $C_1^{(i)}(\omega)$, $C_2^{(i)}(\omega)$ and $C_3^{(i)}(\omega)$, for physical values of masses and mass splittings, are plotted in figures 4, 5 and 6, respectively. Only $C_i^{(5)}(\omega)$ have non-vanishing imaginary parts, as expected from the corresponding marginally allowed decay $D^* \to D\pi$. Of these, only $C_3^{(5)}(\omega)$ has an appreciable imaginary part, which ranges from -0.06 at $\omega = 1$ to -0.04 at $\omega = 1.6$. Numerical values for the real parts of $D_i^{(jB)}$ and $D_i^{(jD)}$ are given in Tables 2 and 3. Only $D_i^{(3D)}$ have nonzero imaginary parts, with values 0.11, -0.40 and -0.75 for $i = 1, 2, 3$, respectively.

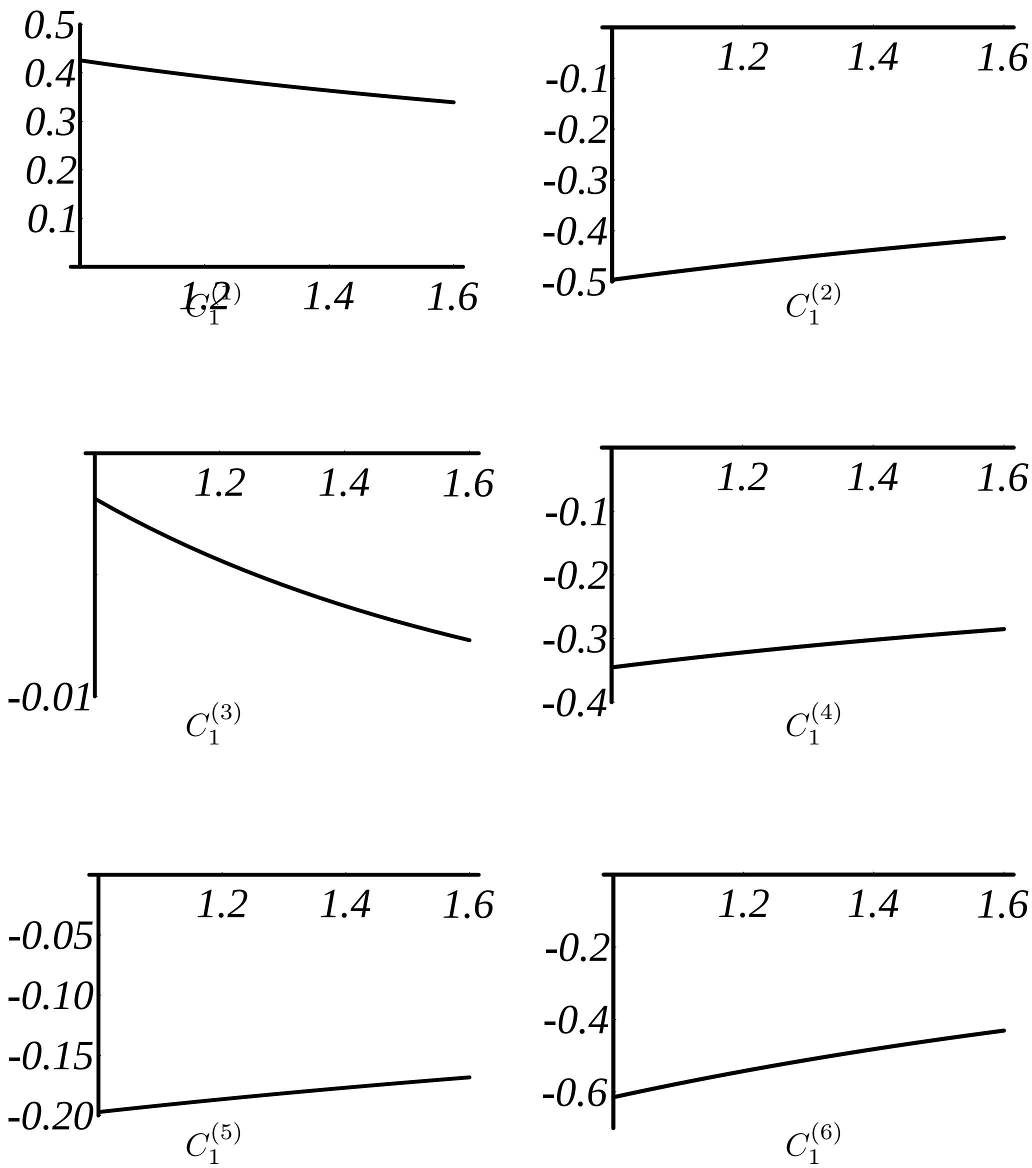

Figure 4. *The real part of the linear combinations of the integral C_1 defined in eq. (B.5), as a function of ω. The integral was done numerically for physical values of the masses.*

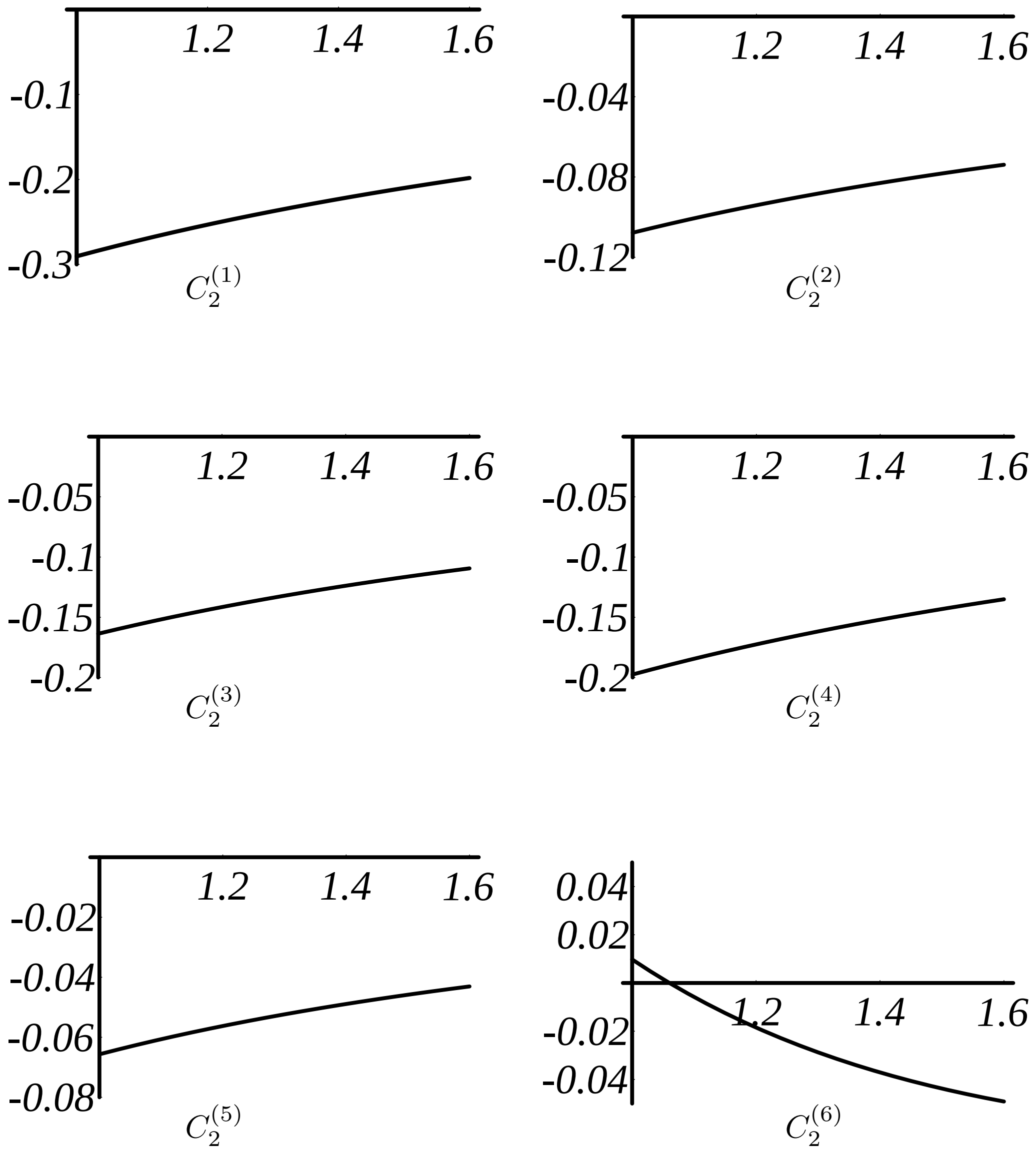

Figure 5. *The real part of the linear combinations of the integral C_2 defined in eq. (B.5), as a function of ω. The integral was done numerically for physical values of the masses.*

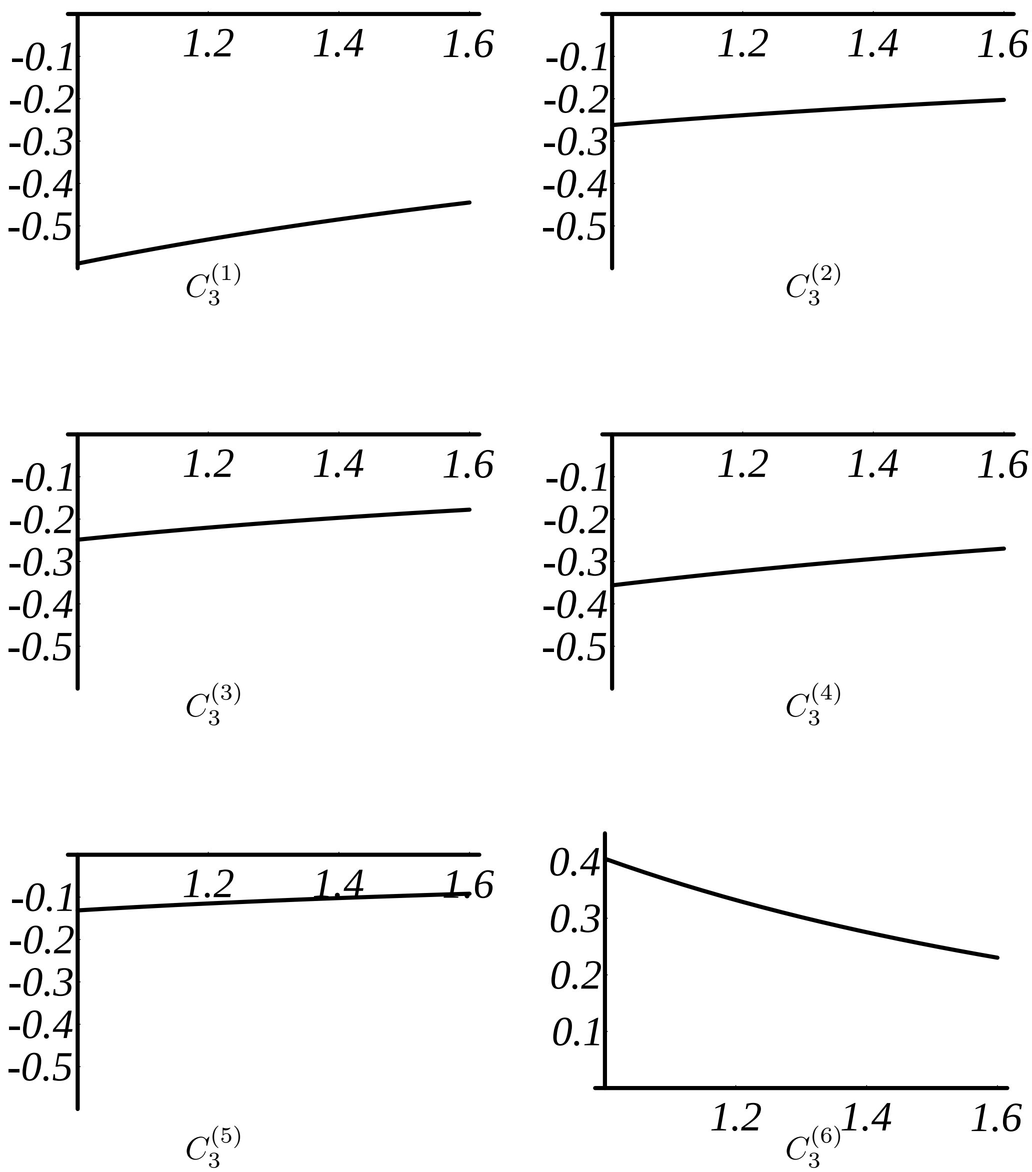

Figure 6. *The real part of the linear combinations of the integral C_3 defined in eq. (B.5), as a function of ω. The integral was done numerically for physical values of the masses.*

Table 2.

i	$D_i^{(1B)}$	$D_i^{(2B)}$	$D_i^{(3B)}$	$D_i^{(4B)}$	$D_i^{(5B)}$	$D_i^{(6B)}$
1	0.139	−0.406	−0.264	−0.517	−0.142	−0.331
2	−0.216	−0.177	−0.055	−0.034	−0.107	−0.184
3	−0.396	−0.319	−0.096	−0.054	−0.191	−0.334
4	−0.396	−0.319	−0.096	−0.054	−0.191	−0.334

Table 3.

i	$D_i^{(1D)}$	$D_i^{(2D)}$	$D_i^{(3D)}$	$D_i^{(4D)}$	$D_i^{(5D)}$	$D_i^{(6D)}$
1	0.696	−0.223	−0.021	−1.144	−0.142	−0.331
2	−0.325	−0.246	−0.109	0.440	−0.107	−0.184
3	−0.614	−0.448	−0.209	0.829	−0.191	−0.334
4	−0.614	−0.448	−0.209	0.829	−0.191	−0.334

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